

The Zakat Formula: A Mathematical Representation of Islamic Charity

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Date: 29 Thu al-Hijjah 1445 AH, 05 July 2024 AD

Zakat, one of the Five Pillars of Islam, is a mandatory form of charity for Muslims who meet specific wealth criteria. It serves as a means of purifying one's wealth and redistributing it to those in need. While the principles of Zakat are well-established, the calculation of the amount due can vary based on the type of assets and their value according to their exchange rates on Zakat time.

This paper focuses on a specific mathematical representation of the Zakat formula using LaTeX, a typesetting system widely used for scientific and mathematical documents.

The Zakat Formula

The Zakat formula can be expressed using the following mathematical equation:

$$Z_{\text{year } n} = \left[P - \sum_{i=1}^{n-1} Z_{\text{year } i} \right] \times Z$$

Where:

- ($Z_{\text{year } n}$): Zakat for the year n (the amount due in a specific year)
- (P): Initial principal (the original amount of wealth subject to Zakat)
- ($\sum_{i=1}^{n-1} Z_{\text{year } i}$): Sum of Zakat paid in previous years (from the first year to the year before the current year, i.e., $n - 1$)
- (Z): Zakat rate (typically 2.5% of the wealth exceeding a certain threshold)

Understanding the Formula

The formula's core idea is to calculate Zakat for a given year based on the remaining wealth after deducting the Zakat paid in previous years. This approach acknowledges that Zakat purifies wealth over time, and the amount payable may decrease annually if the wealth remains relatively stable.

The summation symbol ($\sum_{i=1}^{n-1}$) represents the cumulative Zakat paid from the first year ($i = 1$) to the year preceding the current year ($n - 1$). By subtracting this cumulative Zakat from the initial principal (P), we obtain the remaining wealth subject to Zakat in the current year (n).

Multiplying this remaining wealth by the Zakat rate (Z) gives us the Zakat amount due for the year.

Example

Let's illustrate the formula with an example. Suppose a person has an initial wealth of \$100,000, and the Zakat rate is 2.5%. In the first year, the Zakat due would be:

$$Z_{\text{year } 1} = (100,000 - 0) \times 0.025 = 2,500$$

In the second year, the Zakat due would be:

$$Z_{\text{year } 2} = (100,000 - 2,500) \times 0.025 = 2,437.50$$

This process continues for each subsequent year, with the Zakat amount decreasing gradually as the remaining wealth shrinks.

Rooms & Boxes Methodology

To adapt the Zakat formula for a scenario where transactions are stored in boxes within rooms (analogous to accounts), we need to consider the following modification.

Modified Zakat Formula:

$$Z_{\text{room } r, \text{ year } n} = \left[\sum_{j=1}^m (P_{r,j} - \sum_{i=1}^{n-1} Z_{r,j,i}) \right] \times Z$$

Where:

- ($Z_{\text{room } r, \text{ year } n}$): Zakat for room r in year n
- ($P_{r,j}$): Initial principal in box j of room r
- ($\sum_{j=1}^m$): Summation over all boxes ($j = 1$ to m) in room r
- ($\sum_{i=1}^{n-1} Z_{r,j,i}$): Sum of Zakat paid in previous years for box j in room r (from year 1 to year $n-1$)
- (Z): Zakat rate (typically 2.5%)

Explanation of Modifications:

- Room Index (r): The formula now includes an index ' r ' to represent different rooms (accounts). This allows for calculating Zakat for each room separately.
- Box Index (j): An index ' j ' is introduced to represent different boxes (transactions) within each room. We iterate over all boxes in a room to calculate the total Zakat due for that room.
- Summation over Boxes: We use the summation symbol ($\sum_{j=1}^m$) to add up the Zakat amounts calculated for each box within a room. This gives us the total Zakat due for the entire room.
- Zakat History per Box: The term ($\sum_{i=1}^{n-1} Z_{r,j,i}$) accounts for the Zakat history of each box individually. This ensures that we only calculate Zakat on the remaining balance in each box after deducting previous Zakat payments.

Practical Implementation:

To implement this modified formula, you would typically follow these steps:

- Identify Eligible Rooms/Accounts: Determine which rooms/accounts meet the criteria for Zakat (e.g., minimum wealth threshold).
- Calculate Zakat per Box: For each box in an eligible room, calculate the Zakat due based on the initial principal and previous Zakat payments.
- Aggregate Zakat per Room: Sum up the Zakat amounts for all boxes within a room to get the total Zakat due for that room.
- Repeat for All Rooms: Repeat steps 2 and 3 for all eligible rooms to determine the overall Zakat obligation.

Additional Considerations:

- Timing of Transactions: The formula assumes that all transactions are made at the beginning of the year. If transactions occur throughout the year, you may need to adjust the formula to account for the timing.
- Types of Assets: The formula assumes that all assets are subject to the same Zakat rate. If you have different types of assets with varying Zakat rates, you'll need to calculate Zakat separately for each asset type.

By incorporating these modifications and considerations, you can create a more comprehensive Zakat calculation system that accurately reflects the structure of your accounts and transactions.

Rooms Exchange Rates

Absolutely! To incorporate exchange rates into the Zakat formula for different currencies within each room (account), we can introduce an additional parameter to account for currency conversion.

Enhanced Zakat Formula with Exchange Rates:

$$Z_{\text{room } r, \text{ year } n} = \left[\sum_{j=1}^m (P_{r,j} \times ER_{r,j} - \sum_{i=1}^{n-1} Z_{r,j,i}) \right] \times Z$$

Where:

- $Z_{\text{room } r, \text{ year } n}$: Zakat for room r in year n
- $P_{r,j}$: Initial principal in box j of room r

- $ER_{r,j}$: Exchange rate for the currency in box j of room r (relative to the base currency used for Zakat calculation)
- $\sum_{j=1}^m$: Summation over all boxes ($j = 1$ to m) in room r
- $\sum_{i=1}^{n-1} Z_{r,j,i}$: Sum of Zakat paid in previous years for box j in room r (from year 1 to year $n-1$)
- Z : Zakat rate (typically 2.5%)

Explanation of the Exchange Rate Modification:

The addition of ($ER_{r,j}$) allows us to convert the initial principal in each box to a common base currency. This is essential when dealing with multiple currencies within different accounts. By multiplying the initial principal ($P_{r,j}$) by the corresponding exchange rate ($ER_{r,j}$), we ensure that all amounts are expressed in the same currency for consistent Zakat calculation.

Practical Implementation:

1. **Determine Base Currency:** Choose a base currency for Zakat calculation (e.g., the local currency or a stable international currency).
2. **Obtain Exchange Rates:** Gather the exchange rates for all currencies present in the different boxes/transactions within each room/account.
3. **Convert Principals to Base Currency:** Multiply the initial principal in each box by its corresponding exchange rate to convert it to the base currency.
4. **Apply the Modified Formula:** Use the enhanced Zakat formula to calculate the Zakat due for each box, then aggregate the amounts for each room, and finally, for all eligible rooms.

Example:

Suppose you have a room (account) with two boxes:

- **Box 1:** \$5,000 USD (exchange rate to base currency: 3.75)
- **Box 2:** €2,000 EUR (exchange rate to base currency: 4.20)

After converting to the base currency, the principals become:

- **Box 1:** 5,000 USD * 3.75 = 18,750 base currency units
- **Box 2:** 2,000 EUR * 4.20 = 8,400 base currency units

You would then apply the modified Zakat formula to these converted amounts, along with any previous Zakat payments, to determine the Zakat due for that room.

Important Considerations:

- **Exchange Rate Fluctuations:** Exchange rates can change over time. Choose an appropriate time frame for obtaining exchange rates (e.g., the exchange rate at the beginning of the Zakat year).
- **Zakat on Currency Exchange Gains:** If you have significant gains from currency exchange, you may need to consult with a knowledgeable scholar to determine if Zakat is due on those gains.

By incorporating exchange rates into the Zakat formula, you ensure accurate and fair calculation of Zakat obligations for individuals and institutions holding assets in multiple currencies.

Conclusion

The Zakat formula, as expressed in LaTeX, provides a concise and mathematically rigorous way to calculate Zakat obligations. This formula aligns with the principle that Zakat is a continuous process of purification and redistribution of wealth, aiming to reduce wealth disparities and alleviate poverty within the Muslim community. By understanding and applying this formula, individuals and institutions can ensure that they fulfill their Zakat obligations accurately and contribute to the greater good of society.