

SDS Library Documentation

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First we define an agent, then the functions for the Test Phase, the Diffusion Phase, Halting functions, Iteration functions...

1 Agent

An agent maintains two variables; whether or not it is active and its hypothesis. The Agent class also has a method for initialising a swarm of a given size, all agents are initialised as inactive and with no hypothesis. This necessitates that the Diffusion Phase is run first, such that all agents generate a hypothesis before the test phase.

Defining `__slots__` ensures agents can have no attributes other than those listed, for a slight reduction in memory usage per agent.

5 *⟨agent class definition 5⟩* ≡ (91)

```
class Agent:
    """\
    Data structure for defining an SDS Agent, can only maintain the
    attributes 'hypothesis' and 'active'.\
    """
    __slots__ = ('hypothesis', 'active')

    def __init__(self, hypothesis=None, active=False):
        """\
        Initialise an agent with specific values for hypothesis and active.
        Defaults to inactive with no hypothesis.\
        """
        self.hypothesis = hypothesis

        self.active = active

⟨initialise a list of agents function 6a⟩

⟨agent iterable function 6b⟩
```

Defines:

Agent, used in chunks 6a, 35, 51, 52, 55a, 66, 68–70, 72, 73, 75, 81, 84, and 86.

1.1 Initialisation

6a *<initialise a list of agents function 6a>*≡ (5)

```
@staticmethod
def initialise(agent_count):
    """\
Returns a list of length agent_count, of inactive Agents with no
hypothesis; suitable for use as a swarm. For example:
    swarm = sds.Agent.initialise(agent_count=1000)
    """
    return [
        Agent(hypothesis=None, active=False)
        for _
        in range(agent_count)
    ]
```

Defines:

`initialise`, used in chunks 51, 52, 66, 68–70, 72, 73, 75, 81, 84, and 86.

Uses Agent 5.

Here are two ways to initialise a swarm of 100 agents. Firstly, with the `initialise` function.

```
swarm = Agent.initialise(agent_count=100)
```

Secondly, with a list comprehension.

```
agent_count=100
swarm = [Agent() for _ in range(agent_count)]
```

1.2 Agent iterable function

This makes an Agent iterable, which means the values of an agent can be unpacked with `hypothesis, active = agent`.

6b *<agent iterable function 6b>*≡ (5)

```
def __iter__(self):
    """ Iterating over an agent returns its hypothesis, then its
        activity.
    """
    yield self.hypothesis
    yield self.active
```

Uses activity 50b.

1.3 Agent tuple

It may prove useful to have a minimal implementation of a read only and hashable object for an agent.

7a $\langle \text{agent namedtuple } 7a \rangle \equiv$ (91)

```
ReadOnlyAgent = namedtuple("ReadOnlyAgent", ("hypothesis", "active"))
```

```
ReadOnlyAgent.__doc__ = """\
namedtuple representation of an agent. Attributes are hypothesis and
active.\
"""
```

7b $\langle \text{library dependencies } 7b \rangle \equiv$ (91) 37a▷

```
from collections import namedtuple
```

You can therefore make a read-only copy of a swarm like this.

```
read_only_swarm = [ReadOnlyAgent._make(agent) for agent in swarm]
```

This works because Agent is iterable (See chunk $\langle \text{agent iterable function } 6b \rangle$) and `collections.namedtuple.` maps a sequence into the attributes of a namedtuple.

2 Test phase

2.1 Standard test phase

This is a convenience function, allowing convenient application of the Standard test phase. Standard in this sense refers to a test phase which assumes the agents activities will be set to the value returned from the microtests, i.e. the microtest return values are boolean. Microtests may return integer or float values, but all values will be treated as active, except for 0.

This is a destructive function, modifying the swarm in place.

8 $\langle \text{test phase function 8} \rangle \equiv$

(91)

```
def test_phase(
    swarm,
    microtests,
    multitesting=1,
    multitest_function=all,
    synchronous=True,
    rng=random,
):
    """\
Perform a test phase with boolean microtests.
```

```
This function returns a generator which must be consumed once for each
agent.\
"""
```

```
    test_phase_generator = generic_test_phase(
        swarm=swarm,
        microtests=microtests,
        multitesting=multitesting,
        multitest_function=multitest_function,
        compare=False,
        synchronous=synchronous,
        rng=rng,
    )

    for _ in test_phase_generator:
        yield
```

Defines:

 test_phase, used in chunks 9, 10, 27, 29, 31, 35, 39, and 70.

Uses generic_test_phase 10.

2.2 Comparative test phase

This test phase variant determines the activity of each agent by comparing the score of their test functions with that of a randomly chosen agent.

9 \langle *comparative test phase function 9* $\rangle \equiv$ (91)

```
def comparative_test_phase(  
    swarm,  
    microtests,  
    multitesting=1,  
    multitest_function=max,  
    synchronous=True,  
    rng=random,  
):
```

```
    """\
```

```
    Performs a test phase with scalar microtests, an agent becomes active  
    if their microtest result is larger than that of a randomly chosen  
    agent.
```

```
    This function returns a generator which must be consumed once for each  
    agent.\n    """
```

```
    test_phase = generic_test_phase(  
        swarm=swarm,  
        microtests=microtests,  
        multitesting=multitesting,  
        multitest_function=multitest_function,  
        compare=True,  
        synchronous=synchronous,  
        rng=rng,  
    )
```

```
    for _ in test_phase:  
        yield
```

Defines:

comparative_test_phase, used in chunks 10, 31, 69, and 84.

Uses generic_test_phase 10 and test_phase 8.

2.3 Generic test phase

10 $\langle \text{generic test phase function 10} \rangle \equiv$ (91)

```
def generic_test_phase(
    swarm,
    microtests,
    multitesting=1,
    multitest_function=None,
    compare=False,
    compare_to_boolean=False,
    synchronous=True,
    rng=random,
):
    """\
Perform a test phase. Fully configurable. Consider using the more
convenient and readable functions test_phase and comparative_test_phase.\
"""

    if compare:

        multitest_function = max

    else:

        multitest_function = all

    def make_test(hyp):
        return multitest_function(
            rng.choice(microtests)(hyp)
            for test_num
            in range(multitesting)
        )

    if compare and synchronous:

        test_results = (
            make_test(agent.hypothesis)
            for agent
            in swarm
        )

        test_results = list(test_results)

    if compare_to_boolean:
        test_results = [
            test_result > rng.choice(test_results)
            for test_result
            in test_results
        ]
```

```

        for agent, test_result in zip(swarm, test_results):

            agent.active = test_result
            yield

    else:

        for agent in swarm:

            generic_single_agent_test(
                agent,
                swarm,
                microtests,
                compare,
                compare_to_boolean,
                multitesting,
                multitest_function,
            )

            yield

```

Defines:

 generic_test_phase, used in chunks 8, 9, and 81.

Uses comparative_test_phase 9, generic_single_agent_test 12, and test_phase 8.

2.4 Generic single agent test

Note that in the comparative version, potentially both agents perform a different microtest (or set of microtests if multitesting > 1).

```
12  <generic single agent test function 12>≡ (91)
    def generic_single_agent_test(
        agent,
        swarm,
        microtests,
        compare,
        compare_to_boolean,
        multitesting,
        multitest_function,
        rng=random,
    ):
        """\
        Perform a random microtest, and set the activity, for a single agent.\
        """

        try:
            test_result = multitest_function(
                rng.choice(microtests)(agent.hypothesis)
                for multitest_num
                in range(multitesting)
            )
        except TypeError:
            raise TypeError("Something is None, multitest_function: {mtf}, microtests: {mt}"
                            mtf=multitest_function,
                            mt=microtests,
            ))

        if compare and compare_to_boolean:

            polled_hypothesis = rng.choice(swarm).hypothesis

            polled_result = multitest_function(
                rng.choice(microtests)(polled_hypothesis)
                for multitest_num
                in range(multitesting)
            )

            test_result = test_result > polled_result

        agent.active = test_result
```

Defines:

generic_single_agent_test, used in chunks 10 and 33.
Uses activity 50b.

3 Diffusion phase

These are all destructive functions, modifying the swarm in place.

3.1 Active? Dual?

Can they be done? Bish wants them to be switches separate from context-sensitive and context-free.

3.2 Passive diffusion

This is the diffusion function used in Standard SDS.

13 $\langle \text{passive diffusion function 13} \rangle \equiv$ (91)

```
def passive_diffusion(
    swarm,
    random_hypothesis_function,
    multidiffusion=1,
    rng=random,
):
    """\
Perform a passive diffusion phase.
```

```
This function returns a generator which must be consumed once for each
agent.\
"""
```

```
    diffusion_phase = generic_diffusion(
        swarm,
        random_hypothesis_function,
        context_free=False,
        context_sensitive=False,
        multidiffusion=multidiffusion,
        passive=True,
        active=False,
        rng=rng,
    )

    for _ in diffusion_phase:
        yield
```

Defines:

 passive_diffusion, used in chunks 18, 35, 55b, 58b, 68, 70, 72, 73, 75, 77, 81, and 86.

Uses generic_diffusion 18.

3.3 Context-free diffusion

Similar to passive diffusion, only active agents may become inactive and generate a new hypothesis if they poll an agent which is also active. As active agents may be modified by this process it is necessary to take a snapshot of the state of the swarm before any modification happens, this is the `old_swarm` variable which is populated at the beginning of the phase and never modified.

14 *<context free diffusion function 14>≡* (91)

```
def context_free_diffusion(
    swarm,
    random_hypothesis_function,
    multidiffusion=1,
    passive=True,
    active=False,
    rng=random,
):
    """\
    Perform a context free diffusion phase.
```

```
    This function returns a generator which must be consumed once for each
    agent.\
    """
```

```
        diffusion_phase = generic_diffusion(
            swarm,
            random_hypothesis_function,
            context_free=True,
            context_sensitive=False,
            multidiffusion=multidiffusion,
            passive=passive,
            active=active,
            rng=rng,
        )

        for _ in diffusion_phase:
            yield
```

Defines:

`context_free_diffusion`, used in chunks 18, 31, 56, 77, and 84.

Uses `generic_diffusion` 18.

3.4 Context-sensitive diffusion

Similar to context-free diffusion, only active agents may become inactive and generate a new hypothesis if they poll an agent which is active and both agents share the same hypothesis.

15 $\langle \text{context sensitive diffusion function 15} \rangle \equiv$ (91)

```
def context_sensitive_diffusion(
    swarm,
    random_hypothesis_function,
    multidiffusion=1,
    passive=False,
    active=False,
    rng=random,
):
    """\
Perform a context sensitive diffusion phase.
```

```
This function returns a generator which must be consumed once for each
agent.\
"""
```

```
    diffusion_phase = generic_diffusion(
        swarm,
        random_hypothesis_function,
        context_free=True,
        context_sensitive=True,
        multidiffusion=multidiffusion,
        passive=passive,
        active=active,
        rng=rng,
    )

    for _ in diffusion_phase:
        yield
```

Defines:

context_sensitive_diffusion, used in chunks 18, 31, 57, 73, 77, and 81.

Uses generic_diffusion 18.

3.5 Generic diffusion

The behaviour of passive, context-free and context-sensitive diffusion can be captured in a single, five-variable truth table (Table 1), and then represented as a five-variable Karnaugh map (Table 2). From Table 2 one can derive the logic for Algorithm 1.

CF (E)	CS (D)	a.active (A)	p.active (B)	hyp==hyp (C)	Response
F	F	F	F	F	Random hyp
F	F	F	F	T	Random hyp
F	F	F	T	F	Copy hyp
F	F	F	T	T	Copy hyp
F	F	T	F	F	Maintain hyp
F	F	T	F	T	Maintain hyp
F	F	T	T	F	Maintain hyp
F	F	T	T	T	Maintain hyp
F	T	F	F	F	Don't care
F	T	F	F	T	Don't care
F	T	F	T	F	Don't care
F	T	F	T	T	Don't care
F	T	T	F	F	Don't care
F	T	T	F	T	Don't care
F	T	T	T	F	Don't care
F	T	T	T	T	Don't care
T	F	F	F	F	Random hyp
T	F	F	F	T	Random hyp
T	F	F	T	F	Copy hyp
T	F	F	T	T	Copy hyp
T	F	T	F	F	Maintain hyp
T	F	T	F	T	Maintain hyp
T	F	T	T	F	Random hyp
T	F	T	T	T	Random hyp
T	T	F	F	F	Random hyp
T	T	F	F	T	Random hyp
T	T	F	T	F	Copy hyp
T	T	F	T	T	Copy hyp
T	T	T	F	F	Maintain hyp
T	T	T	F	T	Maintain hyp
T	T	T	T	F	Maintain hyp
T	T	T	T	T	Random hyp

Table 1: The truth table for a combination of passive, context-free and context-sensitive diffusion. CF=isContextFree, CS=isContextSensitive, a.active=agentIsActive, p.active=polledIsActive, hyp==hyp=hypEqualsHyp

Algorithm 1 Generic diffusion

- 1: **if** $\bar{A} \wedge B$ **then**
 - 2: Copy hypothesis (State 2)
 - 3: **else if** $\bar{A} \vee B \wedge E \wedge (C \vee \bar{D})$ **then**
 - 4: Random hypothesis (State 1)
 - 5: **else**
 - 6: Maintain hypothesis (State 3)
-

		\overline{C}	\overline{C}	\overline{C}	\overline{C}	C	C	C	C
		\overline{D}	\overline{D}	D	D	\overline{D}	\overline{D}	D	D
		\overline{E}	E	E	\overline{E}	\overline{E}	E	E	\overline{E}
\overline{A}	\overline{B}	1	1	1	X	1	1	1	X
\overline{A}	B	2	2	2	X	2	2	2	X
A	B	3	1	3	X	3	1	1	X
A	\overline{B}	3	3	3	X	3	3	3	X

Table 2: Five variable Karnaugh map of the diffusion truth table (Table 1). A = agent is active, B = polled agent is active, C = agent and polled agents share a hypothesis, D = context-sensitive diffusion, E = context-free diffusion. 1 is random hypothesis, 2 is copy hypothesis, 3 is maintain hypothesis.

When Context Sensitive or Context Free is being used, then a read-only copy of the swarm must be taken before any of the agents are modified. Otherwise, this function simply performs the diffusion process on each agent.

18 $\langle \text{generic_diffusion function 18} \rangle \equiv$ (91)

```
def generic_diffusion(
    swarm,
    random_hypothesis_function,
    context_free,
    context_sensitive,
    multidiffusion,
    passive,
    active,
    rng=random,
):
    """\
    Perform a diffusion phase, fully configurable. Consider using the more
    convenient and readable functions passive_diffusion,
    context_free_diffusion and context_sensitive_diffusion.

    This function returns a generator which must be consumed once for each
    agent.\
    """

    if context_sensitive:
        context_free = True

    if context_free and not active:
        old_swarm = [ReadOnlyAgent(a.hypothesis,a.active) for a in swarm]
    else:
        old_swarm = swarm

    for agent in swarm:

        generic_single_agent_diffusion(
            agent,
            old_swarm,
            random_hypothesis_function,
            context_free,
            context_sensitive,
            multidiffusion,
            passive,
            active,
            rng,
        )

    yield
```

Defines:

generic_diffusion, used in chunks 13–15.

Uses context_free_diffusion 14, context_sensitive_diffusion 15, generic_single_agent_diffusion 19, and passive_diffusion 13.

3.6 Generic single agent diffusion

```
19  <generic single agent diffusion function 19>≡ (91)
    def generic_single_agent_diffusion(
        agent,
        swarm,
        random_hypothesis_function,
        context_free,
        context_sensitive,
        multidiffusion,
        passive,
        active,
        rng=random,
    ):
        """\
Perform diffusion, and set the hypothesis for a single agent.\
"""
        <handle multidiffusion 20>

        if (
            (active and agent.active and (not polled_agent.active))
            or
            (passive and (not agent.active) and polled_agent.active)
        ):
            if agent.active:
                polled_agent.hypothesis = agent.hypothesis
            else:
                agent.hypothesis = polled_agent.hypothesis
        elif (
            not agent.active
            or
            polled_agent.active
            and context_free
            and (
                not context_sensitive
                or agent.hypothesis == polled_agent.hypothesis
            )
        ):
            agent.active = False
            agent.hypothesis = random_hypothesis_function(random)

Defines:
generic_single_agent_diffusion, used in chunks 18 and 33.
```

3.7 Multidiffusion

The job here is to populate `polled_agent` appropriately, and the way it is done is by setting `polled_agent` to the elements of a random agent generator and stopping when an active agent is reached. If an active agent is never reached then the diffusion will have failed. The `remainder` variable allows for a “real” number of diffusions, where a fraction of a diffusion is a randomly polled agent, followed by a chance of receiving a known inactive agent in its place.

```
20  <handle multidiffusion 20>≡ (19)
    polled_agents = (
        rng.choice(swarm)
        for diffusion_num
        in range(int(multidiffusion))
    )

    for polled_agent in polled_agents:
        if (
            (passive and (not agent.active) and polled_agent.active)
            or (active and agent.active and (not polled_agent.active))
        ):
            break
    else:
        remainder = multidiffusion - int(multidiffusion)

        if remainder > 0:

            if rng.random() > remainder:

                polled_agent = rng.choice(swarm)
```

3.7.1 Probabilistic Rounding Function

The multidiffusion and multitesting functions may benefit from using this probabilistic rounding function. 7.1 gets rounded to 8 with probability 0.1, and rounded to 7 with probability 0.9.

```
21a <probabilistic rounding function 21a>≡
    def probabilistic_round(num, rng=random):
        """\
        Probabilistically round a number, the remainder is the probability of
        rounding up.\
        """

        full = int(num)
        remainder = num - full
        if remainder > 0:
            roundup = rng.random() < remainder
        return full + roundup
```

Defines:

probabilistic_round, never used.

4 Halting functions

4.1 Never halt function

```
21b <never halt function 21b>≡ (91)
    def never_halt(*args, **kwargs):
        """\
        Always returns false, suitable as a halting function for a perpetual
        SDS.\
        """

        return False
```

Defines:

never_halt, used in chunks 35, 39, 70, 75, and 77.

4.2 Stability halt function

22 $\langle \text{stability halt function 22} \rangle \equiv$ (91)

```
def make_stability_halting_function(lower, region, time):
    """\
Returns a function suitable for use as a halting function. Halts, by
returning True when it detects stability, defined by:

lower: Lower bound of proportion of activity of stability window.

region: Amount above lower which defines the upper bound of the
stability window.

time: Number of consecutive times this function must be called with
arguments within the stability window before it will halt.\
"""

    def generator_front_end(activity_count, halt_generator):
        next(halt_generator)
        halted = halt_generator.send(activity_count)
        return halted

    def is_stable_generator(lower, region, time):

        success_count = 0

        while True:

            swarm = yield

            active_count = sum(1 for agent in swarm if agent.active)/len(swarm)

            if active_count < lower or active_count > lower + region:
                success_count = 0
            else:
                success_count += 1

            yield success_count >= time

    halting_generator = is_stable_generator(lower, region, time)

    return functools.partial(
        generator_front_end,
        halt_generator=halting_generator,)
```

Defines:

- `make_stability_halting_function`, used in chunk 77.

Uses activity 50b.

4.3 Instant threshold halt function

23 *<instant threshold halt function 23>*≡ (91)

```
def make_instant_threshold_halt_function(threshold):
    """\
Returns a function suitable for use as a halting function. Halts, by
returning True when the proportion of global activity is greater than
threshold.\
"""
    def threshold_halt_function(swarm, threshold):
        activity = sum(1 for agent in swarm if agent.active)/len(swarm)
        return activity > threshold
    return functools.partial(threshold_halt_function,threshold=threshold)
```

Defines:
 make_instant_threshold_halt_function, used in chunk 77.
Uses activity 50b.

4.4 Threshold time halt function

24 *<threshold time halt function 24>*≡ (91)

```
def make_threshold_time_halting_function(lower, time):
    """\
Returns a function suitable for use as a halting function. Halts, by
returning True when the proportion of global activity is greater than
threshold for a number of calls to this function defined by time.\
"""

    def generator_front_end(activity_count, halt_generator):
        next(halt_generator)
        halted = halt_generator.send(activity_count)
        return halted

    def is_stable_generator(lower, time):

        success_count = 0

        while True:

            swarm = yield

            active_count = sum(1 for agent in swarm if agent.active)/len(swarm)

            if active_count < lower:
                success_count = 0
            else:
                success_count += 1

            yield success_count >= time

    halting_generator = is_stable_generator(lower, time)

    return functools.partial(
        generator_front_end,
        halt_generator=halting_generator,)
```

Defines:

- make_threshold_time_halting_function, never used.

Uses activity 50b.

4.5 Handle halting

25 *<handle halting function 25>*≡ (91)

```
def generic_handle_halting(
    iteration_num,
    swarm,
    halting_iterations,
    halting_function,
    max_iterations,
):

    return (
        (
            halting_iterations
            and iteration_num % halting_iterations == 0
            and halting_function(swarm)
        ) or (
            max_iterations and iteration_num >= max_iterations
        )
    )
```

5 Reporting functions

5.1 Basic report

26a *<basic report function 26a>*≡ (91)

```
def basic_report(
    iteration_num,
    swarm,
    hypothesis_string_function=str,
    max_cluster_report=None,
):

    clusters = count_clusters(swarm)

    agent_count = len(swarm)

    return "{i:4} Activity: {a:0.3f}. {c}".format(
        i=iteration_num,
        a=sum(clusters.values())/float(agent_count),
        c=", ".join(
            "{hyp}:{count}".format(
                hyp=hypothesis_string_function(hyp),
                count=count
            )
            for hyp, count
            in clusters.most_common(max_cluster_report)
        ),
    )

Uses count_clusters 49.
```

5.2 Handle reporting function

26b *<handle reporting function 26b>*≡ (91)

```
def generic_handle_reporting(iteration_num, swarm, report_iterations, report_function):

    if (
        report_iterations
        and iteration_num % report_iterations == 0
    ):
        print(report_function(iteration_num, swarm))
```

6 Iteration functions

There is trouble brewing amongst these iteration functions, they take in one of the convenience test phases, `test_phase` or `comparative_test_phase`, which means the iteration functions don't take the `compare` variable, but they do need to take more extraneous variables like `multitest` and `multitest_function`.

Theoretically the iteration functions should pass the arguments to `generic_test_phase`, but these iteration functions are largely for convenience anyway, so we'll stick with this for now.

As we don't know the value of `compare` we can't determine a sensible default value for `multitest_function`, which is `max` when `compare` is `True` and `all` when `compare` is `False`.

6.1 Single iteration function

This function performs a single Diffusion Phase and a single Test Phase.

```
27  <synchronous iterate function 27>≡ (91)
    def synchronous_iterate(
        swarm,
        microtests,
        random_hypothesis_function,
        diffusion_function,
        test_phase_function=test_phase,
        multidiffusion=1,
        multitesting=1,
        multitest_function=None,
        rng=random,
    ):
        """\
        Performs a synchronous iteration, one diffusion phase for all agents
        followed by one test phase for all agents.

        This function returns a generator which must be consumed once for each
        iteration.\
        """

        while True:

            diffusion_phase_iterator = diffusion_function(
                swarm=swarm,
                random_hypothesis_function=random_hypothesis_function,
                multidiffusion=multidiffusion,
                rng=rng,)

            for _ in diffusion_phase_iterator:
```

```

        pass

    test_phase_iterator = test_phase_function(
        swarm=swarm,
        microtests=microtests,
        multitesting=multitesting,
        multitest_function=multitest_function,
        synchronous=True,
        rng=rng,)

    for _ in test_phase_iterator:
        pass

    yield

```

Defines:

 synchronous_iterate, used in chunks 35 and 51.
 Uses test_phase 8.

6.2 Asynchronous iteration function

This function shuffles the swarm in place, that might not be acceptable eventually, in which case use for agent in random.sample(swarm, len(swarm)).

This function doesn't throw any exceptions, but I've not proved that it's running asynchronously.

I could make this system run more like the parallel implementation, where I call `update_state` rather than relying on the two synchronous phases to proceed in lock step.

29 $\langle \text{asynchronous iterate function 29} \rangle \equiv$ (91)

```
def asynchronous_iterate(
    swarm,
    microtests,
    random_hypothesis_function,
    diffusion_function,
    test_phase_function=test_phase,
    multidiffusion=1,
    multitesting=1,
    multitest_function=None,
    rng=random,
):
    """\
    Performs an asynchronous iteration, all agents are selected in a
    random order to perform one diffusion and one test in turn.

    This function returns a generator which must be consumed once for each
    iteration.\
    """

    while True:

        for agent in swarm:
            if agent.hypothesis is None:
                agent.hypothesis = random_hypothesis_function(random)

        random.shuffle(swarm)

        diffusion_phase_iterator = diffusion_function(
            swarm=swarm,
            random_hypothesis_function=random_hypothesis_function,
            multidiffusion=multidiffusion,
            rng=rng,)

        test_phase_iterator = test_phase_function(
            swarm=swarm,
            microtests=microtests,
            multitesting=multitesting,
            multitest_function=multitest_function,
```

```
        synchronous=False,  
        rng=rng,)  
  
    for _ in zip(diffusion_phase_iterator, test_phase_iterator):  
        pass  
  
    yield  
Defines:  
    asynchronous_iterate, never used.  
Uses test_phase 8.
```

6.3 Parallel iteration function

I might want to check if there's a way of making this run like the asynchronous iteration, where I use the normal test phase and diffusion phase functions and just make them perform randomly.

This is also a bit squirrely, as its taking in functions in the `diffusion_function` parameter, but it's only using them as boolean values because the actual diffusion is performed in `generic_single_agent_diffusion` in `update_state`.

31 $\langle \text{parallel iterate function 31} \rangle \equiv$ (91)

```
def parallel_iterate(
    swarm,
    microtests,
    random_hypothesis_function,
    diffusion_function,
    test_phase_function=test_phase,
    multidiffusion=1,
    passive=True,
    active=False,
    multitesting=1,
    multitest_function=None,
    rng=random,
):
    """\
    Performs a parallel iteration, all agents are updating their state (a
    diffusion followed by a test) in parallel.

    This function returns a generator which simply waits a short time
    between yeilding, this can be used to ensure the parallel process
    runs for a certain amount of wall clock time.\
    """

    context_free = diffusion_function is context_free_diffusion
    context_sensitive = diffusion_function is context_sensitive_diffusion

    compare = test_phase_function is comparative_test_phase
    compare_to_boolean = compare

    if compare:
        multitest_function = max
    else:
        multitest_function = all

    for agent in swarm:
        if agent.hypothesis is None:
            agent.hypothesis = random_hypothesis_function(random)

    worker_threads = (
        threading.Thread(
```

```

        target=update_state,
        args=(
            agent,
            swarm,
            random_hypothesis_function,
            context_free,
            context_sensitive,
            multidiffusion,
            passive,
            active,
            microtests,
            multitesting,
            multitest_function,
            compare,
            compare_to_boolean,
            rng,
        )
    )
    for agent
    in swarm
)

for worker_thread in worker_threads:
    worker_thread.daemon = True
    worker_thread.start()

while True:
    time.sleep(0.01)
    yield

```

Defines:

- parallel_iterate, used in chunk 33.

Uses comparative_test_phase 9, context_free_diffusion 14, context_sensitive_diffusion 15, test_phase 8, and update_state 33.

6.3.1 Update state function

33 $\langle \text{parallel update state function 33} \rangle \equiv$ (91)

```
def update_state(
    agent,
    swarm,
    random_hypothesis_function,
    context_free,
    context_sensitive,
    multidiffusion,
    passive,
    active,
    microtests,
    multitesting,
    multitest_function,
    compare,
    compare_to_boolean,
    rng=random,
):
    """\
Repeatedly perform a diffusion and a test for an agent, with a short
sleep. This function is a helper function to parallel_iterate.\
"""
    while True:

        generic_single_agent_diffusion(
            agent,
            swarm,
            random_hypothesis_function,
            context_free,
            context_sensitive,
            multidiffusion,
            passive,
            active,
            rng,
        )

        generic_single_agent_test(
            agent,
            swarm,
            microtests,
            compare,
            compare_to_boolean,
            multitesting,
            multitest_function,
        )

        sleepy_time = min(2,max(0,random.gauss(1,1)))

        time.sleep(sleepy_time)
```

Defines:

`update_state`, used in chunk 31.

Uses `generic_single_agent_diffusion` 19, `generic_single_agent_test` 12, and `parallel_iterate` 31.

6.4 Run function

This function performs a given number of iterations and returns a list of all the clusters that have formed.

35 `<run function 35>≡` (91)

```
def run(
    swarm,
    microtests,
    random_hypothesis_function,
    max_iterations=1000,
    diffusion_function=passive_diffusion,
    halting_function=never_halt,
    halting_iterations=1,
    multitesting=1,
    multitest_function=None,
    report_iterations=10,
    test_phase_function=test_phase,
    hypothesis_string_function=str,
    max_cluster_report=None,
    iteration_function=synchronous_iterate,
    multidiffusion=1,
    rng=random,
):
    """\
```

The main front end to the SDS library. This will run forever until manually halted, or until a halting condition is reached, afterwhich it will return a collections.Counter of all the clusters.

```
:param swarm: A list of sds.Agent instances.
:param microtests: A collection of functions all of which take a hypothesis\
, as returned by random_hypothesis_function and return the result of a\
microtest as either a scalar value or a boolean. All tests must \
return the same type.
:param random_hypothesis_function: A function which takes a random number \
generator and returns hypothesis suitable as input for all the \
functions in microtests.
:param max_iterations: The number of iterations afterwhich the SDS will \
halt. If max_iterations=None, the SDS will never halt due to the num\
ber of iterations, but may be manually halted, or halted by the halti\
ng function.
:param diffusion_function: The diffusion function to use in the diffusion \
phase.
:param random: A random number generator, probably an instance of the rand\
om.Random class. Use an instance with an explicit seed to get repeata\
ble behaviour, else you may pass in the random module itself.
:param halting_function: (Default: never_halt) The function which takes \
the swarm as input and returns True if its condition is met.
:param halting_iterations: (Default: 0) The number of iterations between \
each call of the halting_function. If halting_iterations is a Falsy \
```

```

value (e.g. None, 0, False, [], '') then the halting_function is \
never called.
:param multitesting: (Default: 1) The number of microtests each agent \
performs in the test phase. Must be an integer.
:param multitest_function: (Default: None) The function which takes a \
list of microtest results and turns them into a single result. The \
most likely values for multitest_function will be 'all' i.e. all \
microtests must pass, and 'any' i.e. at least one microtest must pass.
:param report_iterations: (Default: None) The number of iterations \
between each report to stdout of the hypotheses with the largest \
clusters.
:param test_phase_function: (Default: test_phase) The function to use in \
the test phase.
:param hypothesis_string_function: (Default: str) The function to call \
on a hypothesis to turn it into a string, suitable for inclusion in \
the report. If hypotheses are using built-in data types, str is \
often enough, otherwise a custom 'to string' function must be supplied.
:param max_cluster_report: (Default: None) The maximum number of \
clusters to include in the report.
:param iteration_function: (Default: synchronous_iterate) The iteration \
function to call once per iteration.
:param multidiffusion: (Default: 1) The number of agents for a polling \
agent to poll during the diffusion phase. May be an integer or a \
float.\
"""

```

```

    report_function = functools.partial(
        basic_report,
        hypothesis_string_function=hypothesis_string_function,
        max_cluster_report=max_cluster_report,
    )

```

```

    handle_reporting = functools.partial(
        generic_handle_reporting,
        report_iterations=report_iterations,
        report_function=report_function,
    )

```

```

    handle_halting = functools.partial(
        generic_handle_halting,
        halting_iterations=halting_iterations,
        halting_function=halting_function,
        max_iterations=max_iterations,
    )

```

```

    try:

```

```

        iteration_generator = iteration_function(
            swarm=swarm,

```

```

        microtests=microtests,
        random_hypothesis_function=random_hypothesis_function,
        diffusion_function=diffusion_function,
        test_phase_function=test_phase_function,
        multidiffusion=multidiffusion,
        multitesting=multitesting,
        multitest_function=multitest_function,
        rng=rng,
    )

    for iteration_num, iteration in enumerate(iteration_generator):

        handle_reporting(iteration_num, swarm)

        if handle_halting(iteration_num, swarm):

            break

    except KeyboardInterrupt:

        pass

    return count_clusters(swarm)

```

Defines:

- `run`, used in chunks 39, 41b, 42, 45–48, 52, 68, 69, 72, 75, 81, 84, and 86.
- Uses Agent 5, `count_clusters` 49, `never_halt` 21b, `passive_diffusion` 13, `synchronous_iterate` 27, and `test_phase` 8.

37a $\langle \text{library dependencies } 7b \rangle + \equiv$ (91) $\langle 7b \ 38 \rangle$
 `import itertools`

6.5 Write swarm to file

37b $\langle \text{write swarm function } 37b \rangle \equiv$ (91)
 `def write_swarm(swarm, outfile):`
 `"""`
 Writes a swarm to a file-like object.
 `"""`
 `json.dump(`
 `{`
 `'agent count':len(swarm),`
 `'clusters':count_clusters(swarm).most_common(),`
 `},`
 `outfile,`
 `)`

Defines:

- `write_swarm`, used in chunk 39.
- Uses `count_clusters` 49.

38 $\langle \textit{library dependencies 7b} \rangle + \equiv$
 `import json`

(91) $\triangleleft 37a \ 41a \triangleright$

6.6 Continuous loop function

39 *<run daemon 39>*≡ (91)

```
def run_daemon(
    swarm,
    microtests,
    random_hypothesis_function,
    diffusion_function,
    max_iterations=None,
    halting_function=never_halt,
    halting_iterations=0,
    multitesting=1,
    multitest_function=None,
    report_iterations=None,
    test_phase_function=test_phase,
    hypothesis_string_function=str,
    max_cluster_report=None,
    out_file_name='/tmp/clusters.json',
    rng=random,
):
    """\
Calls sds.run in a daemon thread. The daemon can be interacted with
through the command line.

q: Halt the SDS and kill the daemon.
c: Print the largest clusters to the screen.
w: Write the largest clusters to file.

Anything else is printed to stdout.\
"""

    def write_status(swarm):
        with open(out_file_name, 'w') as f:
            write_swarm(swarm, f)
            print('wrote swarm status to', out_file_name)

    control_queue = queue.Queue()
    control_queue.put(True)

    def swarm_iterator():
        print('starting SDS')
        run(
            swarm,
            microtests,
            random_hypothesis_function,
            max_iterations,
            diffusion_function,
            halting_function,
            halting_iterations,
```

```

        multitesting,
        multitest_function,
        report_iterations,
        test_phase_function,
        hypothesis_string_function,
        max_cluster_report,
        rng,)

    print('finishing SDS')

    write_status(swarm)

    control_queue.task_done()

t = threading.Thread(target=swarm_iterator)
t.daemon = True # Program will exit when only daemons are left.
t.start()
del t

def interface_manager():

    while True:

        instr = input()

        if instr == 'q':
            print("'q' received quitting")
            control_queue.task_done()
            break
        elif instr == 'c':
            print(count_clusters(swarm).most_common(max_cluster_report))
        elif instr == 'w':
            write_status(swarm)
        else:
            print('You said:',instr.upper())

t = threading.Thread(target=interface_manager)
t.daemon = True # Program will exit when only daemons are left.
t.start()
del t

control_queue.join()

print('done run_daemon')

return count_clusters(swarm)

```

Defines:

run_daemon, used in chunk 70.

Uses count_clusters 49, never_halt 21b, run 35, test_phase 8, and write_swarm 37b.

41a $\langle \text{library dependencies 7b} \rangle + \equiv$ (91) $\triangleleft 38 \ 50a \triangleright$

```

import queue
import time # for time.sleep
import itertools # for itertools.count
import threading # for threading.Thread

```

7 Coupled SDS

7.1 Coupled diffusion

41b $\langle \text{coupled diffusion 41b} \rangle \equiv$ (91)

```

def coupled_diffusion(
    swarms,
    random,
    random_hypothesis_functions,
    diffusion_functions,
):
    """\
    Performs Coupled diffusion when passed a list of swarms, a list of
    random hypothesis functions and a list of diffusion functions. Not
    tested with the newest version of sds.run.\
    """
    for swarm, diffusion_function, random_hypothesis_function in zip(
        swarms,
        diffusion_functions,
        random_hypothesis_functions
    ):
        diffusion_function(swarm, random, random_hypothesis_function)

```

Defines:

 coupled_diffusion, used in chunk 47.

Uses run 35.

7.2 Coupled test phase

Multitesting is weird for a coupled test phase, as each test will involve more random agents, so after ten tests you've hit at least ten agents, and it would be strange to set them all inactive if a couple were good, or active if lots were bad.

This is of course not weird if you have a component hypothesis, and a data swarm, and a number of partial evaluations, then multitesting would be to select a hypothesis, and a data point, and to test it against multiple microtests. It's possible that it only seems strange because in hyperplane parameter estimation there is only one test. An alternative is to perform the test with the hypothesis and different data points, and to update the data point with the result of each test, but only update the hypothesis with the result of the combined tests.

Comparative testing can be done when multitesting equals 1, but I'll have to record the randomly selected agents for each master agent.

Now, is it the case that you don't need multitesting if you have coupled sds? Multitesting effectively increases or decreases the test score, but coupled sds weights the test selection.

Maybe there's a decision to be made, if I had multitesting equal to 10, and that meant two hypothesis agents and two data agents, I'd affect the activity of 31 agents. So which of these options actually makes sense? The activity of the test which made the change is impossible as in the "all" case or "any" case I can't tell which one is responsible.

I think the activity of the master agent should be pushed onto all of the other agents queried in the multitesting. But I don't think I can be arsed to implement it now.

```
42  <coupled test phase 42>≡ (91)
    def generic_coupled_test_phase(
        master_swarm_num,
        swarms,
        random,
        multitesting,
        multitest_function,
        microtests,
        compare,
    ):
        """\
        Performs Coupled test when passed the index of a master swarm, a list
        of swarms, and a list of lists of microtests. Not tested with the
        latest version of sds.run.\
        """

        if not multitesting == 1:
            raise NotImplementedError(
                "Sorry, I've not got around to multitesting for coupled "
                "sds yet. When I do, remember to make multitest_function"
                " default to None, as it should default to 'all' when sc"
                "alar=True and 'max' when compare=False")
```

```

test_results = []

tested_agents = []

for master_agent in swarms[master_swarm_num]:

    agents = [random.choice(swarm) for swarm in swarms]

    agents[master_swarm_num] = master_agent

    hypotheses = tuple(agent.hypothesis for agent in agents)

    microtest = random.choice(microtests)

    test_results.append(microtest(*hypotheses))

    tested_agents.append(agents)

if False and compare:

    test_results = [
        test_result > random.choice(test_results)
        for test_result
        in test_results
    ]

for test_result, agents in zip(test_results, tested_agents):

    for agent in agents:

        agent.active = test_result

```

Defines:

 generic_coupled_test_phase, used in chunks 45 and 46.

Uses run 35.

This is the abandoned code chunk for coupled sds multitesting.

```
44  <coupled multitesting 44>≡
    test_results = []

    for test_num in range(multitesting):

        agents = [random.choice(swarm) for swarm in swarms]

        agents[master_swarm_num] = master_agent

        hypotheses = tuple(agent.hypothesis for agent in agents)

        microtest = random.choice(microtests)

        test_results.append(microtest(*hypotheses))

    result = multitest_function(test_results)

    for agent in agents:

        agent.active = result
```

7.2.1 Synchronous coupled test phase

45 *<synchronous coupled test phase 45>*≡ (91)

```
# master/slave synchronisation
def synchronous_coupled_test_phase(
    swarms,
    random,
    multitesting,
    multitest_function,
    microtests,
    compare,
):
    """\
    Perform a Synchronous coupled test phase. Not tested with the latest
    version of sds.run.\
    """

    master_swarm_num = 0

    generic_coupled_test_phase(
        master_swarm_num,
        swarms,
        random,
        multitesting,
        multitest_function,
        microtests,
        compare,
    )
```

Defines:

synchronous_coupled_test_phase, used in chunk 73.

Uses generic_coupled_test_phase 42 and run 35.

7.2.2 Sequential coupled test phase

46 \langle *sequential coupled test phase 46* $\rangle \equiv$ (91)

```
def sequential_coupled_test_phase(
    swarms,
    random,
    multitesting,
    multitest_function,
    microtests,
    compare,
):
    """\
    Perform a Sequential master coupled test phase. Not tested with the latest
    version of sds.run.\
    """

    for master_swarm_num in range(len(swarms)):

        generic_coupled_test_phase(
            master_swarm_num,
            swarms,
            random,
            multitesting,
            multitest_function,
            microtests,
            compare,
        )
```

Defines:

- `sequential_coupled_test_phase`, used in chunk 73.

Uses `generic_coupled_test_phase` 42 and run 35.

7.3 Coupled iteration

47 $\langle \text{coupled iterate 47} \rangle \equiv$ (91)

```
def iterate_coupled(
    swarms,
    random_hypothesis_functions,
    diffusion_functions,
    random,
    multitesting,
    multitest_function,
    report_iterations,
    test_phase_function,
    microtests,
    compare,
):
    """\
    Perform an iteration of Coupled SDS. Not tested with the latest
    version of sds.run.\
    """

    coupled_diffusion(
        swarms,
        random,
        random_hypothesis_functions,
        diffusion_functions)

    test_phase_function(
        swarms,
        random,
        multitesting,
        multitest_function,
        microtests,
        compare,
    )
```

Defines:

 iterate_coupled, used in chunk 48.

Uses coupled_diffusion 41b and run 35.

```

def run_coupled(
    swarms,
    random_hypothesis_functions,
    max_iterations,
    diffusion_functions,
    random,
    multitesting,
    multitest_function,
    report_iterations,
    test_phase_function,
    hypothesis_string_function,
    max_cluster_report,
    microtests,
    compare,
):
    """\
Perform a Coupled SDS. Not tested with the latest version of sds.run.\
"""

    if max_iterations is None:

        iterator = itertools.count()

    else:

        iterator = range(max_iterations)

    if compare:

        multitest_function = max

    else:

        multitest_function = all

    try:

        for iteration in iterator:

            iterate_coupled(
                swarms,
                random_hypothesis_functions,
                diffusion_functions,
                random,
                multitesting,
                multitest_function,
                report_iterations,
                test_phase_function,

```



```

        microtests,
        compare,
    )

    if report_iterations and iteration % report_iterations == 0:

        clusters_list = tuple(count_clusters(swarm) for swarm in swarms)

        agent_counts = tuple(len(swarm) for swarm in swarms)

        active_count = tuple(
            sum(clusters.values())
            for clusters
            in clusters_list)

        print(agent_counts, active_count, clusters_list)

    except KeyboardInterrupt:

        pass

    return tuple(count_clusters(swarm) for swarm in swarms)

```

Defines:
 run_coupled, used in chunk 73.
 Uses count_clusters 49, iterate_coupled 47, and run 35.

8 Analysis functions

8.1 Count clusters function

This function returns a list of all the clusters in the swarm.

```

49  <count clusters function 49>≡ (91)
    def count_clusters(swarm):
        """\
        Returns the number of active agents at each hypothesis with at least one
        active agent as a collections.Counter.\
        """

        return collections.Counter(
            agent.hypothesis
            for agent
            in swarm
            if agent.active
        )

```

Defines:
 count_clusters, used in chunks 26a, 35, 37b, 39, 48, and 64.

50a $\langle \text{library dependencies 7b} \rangle + \equiv$
import collections

(91) $\langle 41a \ 61b \rangle$

8.2 Calculate Activity

50b $\langle \text{activity function 50b} \rangle \equiv$ (91)
def activity(swarm):
 """\
 Return the proportion of the swarm which are active between 0 and 1.\
 """

 agent_count = len(swarm)

 active_count = sum(1 for agent in swarm if agent.active)

 return active_count/agent_count

Defines:

activity, used in chunks 6b, 12, 22–24, and 51.

8.3 Estimate Uniform Background Noise

51 $\langle \text{estimate noise 51} \rangle \equiv$ (91)

```
def estimate_noise(
    microtests,
    random_hypothesis_function,
    noise_agent_count=100,
    iterations=100,
):
    """\
Returns an estimate of the uniform background noise, between 0 and 1.\
"""

    def no_diffusion(swarm, random, random_hypothesis_function):
        for agent in swarm:
            agent.active = False
            agent.hypothesis = random_hypothesis_function(random)

    noise_swarm = Agent.initialise(100)

    activities = []

    for iteration in range(iterations):
        synchronous_iterate(
            noise_swarm,
            microtests,
            random_hypothesis_function,
            no_diffusion,
            random,)

        activities.append(activity(noise_swarm))

    return sum(activities)/iterations
```

Defines:

`estimate_noise`, never used.

Uses activity 50b, Agent 5, initialise 6a, and synchronous_iterate 27.

8.4 Swarm from clusters

A full swarm can be recovered from a clusters object and a count of the total number of agents.

```
52  <swarm from clusters 52>≡ (91)
    def swarm_from_clusters(agent_count, clusters):
        """\
        Returns a swarm suitable for use in functions like sds.run.

        Clusters should be a dictionary or collections.Counter of the
        hypotheses of active agents.\
        """

        active_agents = (
            (
                Agent(hypothesis=hyp, active=True)
                for _ in
                range(count)
            )
            for hyp, count
            in clusters.items())

        inactive_count = agent_count - sum(clusters.values())

        return (
            Agent.initialise(inactive_count)
            + list(itertools.chain.from_iterable(active_agents)))
```

Defines:

swarm_from_clusters, never used.

Uses Agent 5, initialise 6a, and run 35.

8.5 Pretty printing

This function renders a list of clusters into text.

```
53  <pretty print clusters with values 53>≡ (91)
    def pretty_print_with_values(clusters, search_space, max_clusters=None):

        string_template = "{c:6d} at hyp {h:6d} (value: {e:0.6f})"

        cluster_strings = [
            string_template.format(
                c=count,
                h=hyp,
                e=search_space[hyp])
            for hyp, count
            in clusters.most_common(max_clusters)
        ]

        return "\n".join(cluster_strings)
```

Defines:

pretty_print_with_values, never used.

9 Multilayer SDS

9.1 Swarm class

Multilayer SDS mostly uses functions which are very similar to Standard SDS, but a Swarm class is also required to associate each swarm of agents with a particular function for generating a random hypothesis.

```
54  <swarm class 54>≡ (91)
    class Swarm:
        """
        A multilayer SDS swarm. Best instansiated with make_ml_sds."""

        <swarm initialisation function 55a>

        <swarm passive diffusion function 55b>

        <swarm context free diffusion function 56>

        <swarm context sensitive diffusion function 57>

        <swarm test function 58a>

        <swarm iterate function 58b>

        <swarm set activity function 59b>

        <swarm set hypothesis function 59a>
    Uses make_ml_sds 60.
```

Swarm initialisation function

```
55a  <swarm initialisation function 55a>≡ (54)
      def __init__(self, size, random_hypothesis_function, lower_layer=None):

          self.agents = [
              Agent(active=False, hypothesis=None)
              for _
              in range(size)
          ]

          if lower_layer is None:

              lower_layer = []

          self.lower_layer = lower_layer

          self.random_hypothesis = random_hypothesis_function
```

Uses Agent 5.

Swarm diffusion function This implements passive diffusion for Multilayer SDS. It's not clear whether this could be modified to be identical to the functions written for SDS, or if the other Diffusion Phase variants need to be reimplemented for Multilayer SDS.

```
55b  <swarm passive diffusion function 55b>≡ (54)
      @staticmethod
      def passive_diffusion(swarm, solitariness, random):

          for agent_num, agent in enumerate(swarm.agents):

              if not agent.active:

                  polled_agent = random.choice(swarm.agents)

                  if polled_agent.active and random.random() > solitariness:

                      swarm.set_hypothesis(agent_num, polled_agent.hypothesis)

              else:

                  agent.hypothesis = swarm.random_hypothesis(
                      agent_num,
                      random,
                  )
```

Uses passive_diffusion 13.

56 $\langle \text{swarm context free diffusion function 56} \rangle \equiv$ (54)

```
@staticmethod
def context_free_diffusion(swarm, random):

    for agent_num, agent in enumerate(swarm.agents):

        polled_agent = random.choice(swarm.agents)

        if agent.active:

            if polled_agent.active:

                swarm.set_activity(agent_num, False)

                agent.hypothesis = swarm.random_hypothesis(
                    agent_num,
                    random,
                )

            else:

                if polled_agent.active:

                    swarm.set_hypothesis(agent_num, polled_agent.hypothesis)

                else:

                    agent.hypothesis = swarm.random_hypothesis(
                        agent_num,
                        random,
                    )
```

Uses context_free_diffusion 14.


```

    @staticmethod
    def context_sensitive_diffusion(swarm, random):

        for agent_num, agent in enumerate(swarm.agents):

            polled_agent = random.choice(swarm.agents)

            if agent.active:

                if (
                    polled_agent.active
                    and (agent.hypothesis == polled_agent.hypothesis)
                ):

                    swarm.set_activity(agent_num, False)

                    agent.hypothesis = swarm.random_hypothesis(
                        agent_num,
                        random,
                    )

            else:

                if polled_agent.active:

                    swarm.set_hypothesis(agent_num, polled_agent.hypothesis)

                else:

                    agent.hypothesis = swarm.random_hypothesis(
                        agent_num,
                        random,
                    )

```

Uses context_sensitive_diffusion 15.

Swarm test function The test function is similar to the standard Test Phase only the result of the test is propagated recursively to all connected agents.

58a $\langle \text{swarm test function 58a} \rangle \equiv$ (54)

```

def test(self, microtests, random, multitest=1, multitest_fun=None):

    for num, agent in enumerate(self.agents):

        microtest = random.choice(microtests)

        if multitest == 1:

            self.set_activity(num, microtest(agent.hypothesis))

        else:

            self.set_activity(
                num,
                multitest_fun(
                    random.choice(microtests)(agent.hypothesis)
                    for _
                    in range(multitest)
                )
            )

```

Swarm iterate function This is a convenience function which calls one diffusion phase followed by one test phase.

58b $\langle \text{swarm iterate function 58b} \rangle \equiv$ (54)

```

def iterate(
    self,
    microtests,
    random,
    diffusion_function=passive_diffusion.__func__,
    multitest=1,
    multitest_fun=None,
    solitariness=1,
):
    diffusion_function(self, solitariness, random)
    self.test(microtests, random, multitest, multitest_fun)

```

Uses passive_diffusion 13.

Swarm set hypothesis function

59a $\langle \text{swarm set hypothesis function 59a} \rangle \equiv$ (54)

```
def set_hypothesis(self, agent_num, new_hypothesis):  
  
    self.agents[agent_num].hypothesis = new_hypothesis  
  
    if len(self.lower_layer) == 0:  
        return  
  
    for lower_swarm, hypothesis_component in (  
        zip(self.lower_layer, new_hypothesis)):  
  
        lower_swarm.set_hypothesis(agent_num, hypothesis_component)
```

Swarm set activity function

59b $\langle \text{swarm set activity function 59b} \rangle \equiv$ (54)

```
def set_activity(self, agent_num, new_activity):  
  
    self.agents[agent_num].active = new_activity  
  
    for swarm in self.lower_layer:  
  
        swarm.set_activity(agent_num, new_activity)
```

9.2 Multilayer SDS factory function

To properly implement Multilayer SDS the random hypothesis generation functions recursively call the random hypothesis generation functions of lower layers, and so a convenience function has been developed that takes an intended topology of swarms and composes all the required random hypothesis generation functions.

```
60  <make multilayer sds function 60>≡ (91)
    def make_ml_sds(swarm_size, bottom_hyp_functions, topology):

        lower_layer = [
            Swarm(
                size=swarm_size,
                random_hypothesis_function=hyp_fun
            )
            for hyp_fun
            in bottom_hyp_functions
        ]

        for layer_num, swarm_splits in enumerate(topology, start=1):

            <construct new layer 61a>

            lower_layer = layer

            top_swarm = lower_layer[0]

        return top_swarm
```

Defines:

`make_ml_sds`, used in chunks 54 and 64.

61a	<pre> <construct new layer 61a>≡ layer = [] swarm_offset = 0 for split_num, swarm_split in enumerate(swarm_splits): lower_layer_start = swarm_offset lower_layer_end = swarm_offset+swarm_split random_hypothesis_function = functools.partial(random_compound_hyp, lower_layer[lower_layer_start:lower_layer_end],) new_swarm = Swarm(size=swarm_size, lower_layer=lower_layer[lower_layer_start:lower_layer_end], random_hypothesis_function=random_hypothesis_function) layer.append(new_swarm) swarm_offset += swarm_split </pre> <p>Uses random_compound_hyp 62b.</p>	(60)
61b	<pre> <library dependencies 7b>+≡ import functools </pre>	(91) <50a 63b>

9.3 Random hypothesis, and single diffusion functions

Multilayer SDS requires that a single agent can perform the diffusion process, not the whole swarm, this is the purpose of the `single_diffusion` function. It also requires that agents in higher level swarms can randomly generate a hypothesis by calling the diffusion function on each of the associated agents in the swarms in the lower layer, this is the purpose of the `random_compound_hyp` function.

Single diffusion function

62a *<single diffusion function 62a>*≡ (91)

```
def single_diffusion(agent_num, swarm, random):

    # agent is guaranteed to be inactive
    agent = swarm.agents[agent_num]

    polled_agent = random.choice(swarm.agents)

    if polled_agent.active:

        swarm.set_hypothesis(agent_num,polled_agent.hypothesis)

        return polled_agent.hypothesis

    else:

        new_hyp = swarm.random_hypothesis(agent_num, random)

        agent.hypothesis = new_hyp

        return new_hyp
```

Defines:

`single_diffusion`, used in chunk 62b.

Random compound hypothesis function

62b *<random compound hypothesis function 62b>*≡ (91)

```
def random_compound_hyp(lower_swarms, num, random):

    return tuple(
        single_diffusion(num, lower_swarm, random)
        for lower_swarm
        in lower_swarms
    )
```

Defines:

`random_compound_hyp`, used in chunk 61a.

Uses `single_diffusion` 62a.

9.4 Flatten hypothesis function

Hypotheses generated by Multilayer SDS can be very deeply nested lists, this function allows the removal of some of the levels of nesting, until a single flat list is returned. A flat list will be returned if the value if times=len(topology)-1.

```
63a <flatten hypothesis 63a>≡ (91)
    def flatten_hypothesis(hypothesis,times):
        new_hypothesis = itertools.chain.from_iterable(hypothesis)
        if times == 1:
            return list(new_hypothesis)
        else:
            return flatten_hypothesis(new_hypothesis,times-1)

63b <library dependencies 7b>+≡ (91) <61b 76>
    import itertools
```

9.5 Concurrent Execution

Python has some libraries for concurrent execution, this investigation is to determine if SDS can be executed concurrently to improve performance.

This is not an investigation into the absolute performance itself, as Python is not the optimal language for speed of execution, simply to discover which architectures, if any, offer significant performance improvements.

Python has different methods for when the task is limited by speed of execution (CPU bound), or speed of data transfer (I/O bound). If it's CPU bound you need to use multi-processing, multithreading will work fine for I/O bound tasks.

"The strength of threads is shared state. The weakness of threads is shared state (managing race conditions)." From https://dl.dropboxusercontent.com/u/3967849/pyru/_build/html/int Thinking about Concurrency, Raymond Hettinger, Python core developer.

Some tasks will be soluable within a single thread. More complex problems will need multiple cores, truly large problems need distributed processing. Second, "multiple cores" category is becoming less relevant in this world. Don't spend too much time on it, if the real issue is in much larger problems.

When you have threads that end, you join the threads, when you have deamon threads which never terminate, you join the message queue.

Queues are only suitable for when your underlying data flow is a directed acyclic graph.

A good development strategy is to first use "map" to test the code in a single process and a single thread before moving to parallel execution.

9.6 MSDS Example

```
64  <multilayer sds example 64>≡
    import sds # Import sds library
    import random
    r = random.Random() # Initialise random number generator

    # Initialise a list of one function per swarm, they take agent_num, and
    # random, and return a hypothesis component.
    random_hyp_functions = [
        lambda num, random: random.randrange(10),
        lambda num, random: random.randrange(10),
        lambda num, random: random.randrange(10),
        lambda num, random: random.randrange(10),
    ]

    # Declare a topology, starting with a list of one [[1]] for each swarm,
    # make a list of lists of ints, where each list of ints means how the
    # previous numbers will be divided.
    topology = [[2,2],[2]]

    # Create a top swarm, with all its related infrastructure.
    swarm = sds.make_ml_sds(
        swarm_size=100,
        bottom_hyp_functions=random_hyp_functions,
        topology=topology,
    )

    <initialise search space 65a>
    <initialise microtests 65b>

    # Iterate the swarm 100 times.
    for _ in range(100):
        swarm.iterate(
            microtests=microtests,
            random=r,
        )

    # You can use the free count_clusters function on Swarm.agents.
    clusters = sds.count_clusters(swarm.agents)

    print(clusters.most_common(10))
    Uses count_clusters 49 and make_ml_sds 60.
```


The search space is a 4 dimensional array of random numbers in the interval $[-1, 1]$.

65a $\langle \text{initialise search space 65a} \rangle \equiv$ (64)

```

search_space = [
    [
        [
            r.uniform(-1,1)
            for _
            in range(10)
        ]
        for _
        in range(10)
    ]
    for _
    in range(10)
]

```

A list of microtests, they take a hyp, and return a boolean. This isn't a member variable of Swarm as only the top swarm is involved in testing.

65b $\langle \text{initialise microtests 65b} \rangle \equiv$ (64)

```

microtests = [
    lambda h: True,
    lambda h: False,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.1,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.2,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.3,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.4,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.5,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.6,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.7,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.8,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 0.9,
    lambda h: search_space[h[0][0]][h[0][1]][h[1][0]][h[1][1]] > 1,
]

```

10 Multilater SDS implemented with the library

This is distinct from the deprecated Class-based version.

```
66  <src/multilayer-sds.py 66>≡
    import random
    import functools
    import sds

    search_space = [
        [1,2,3],
        [4,5,6],
        [7,8,9],
    ]

    def random_row(rng=random):
        return rng.randrange(len(search_space))

    row_swarm = sds.Agent.initialise(agent_count=100)

    def random_col(rng=random):
        return rng.randrange(len(search_space[0]))

    col_swarm = sds.Agent.initialise(agent_count=100)

    def new_hyp(lower_agents, new_hyp_functions, rng=random):
        """\
        If lower agents are active then take their hyp, else make the
        generate a random hyp.\
        """
        return [
            new_hyp_function(lower_agent)
            for lower_agent, new_hyp_function
            in zip(lower_agents, new_hyp_functions)]

    def make_ml_swarm(layers, agent_count):

        ml_swarm = []

        for agent_num in range(agent_count):

            agent = MultilayerAgent(
                probes=[layer[agent_num] for layer, new_hyp_fun in layers],
                new_hyp_functions=[new_hyp_fun for layer, new_hyp_fun in layers],
                active=False,
                hypothesis=None,)

            ml_swarm.append(agent)
```

```

        return ml_swarm

class MultilayerAgent:
    def __init__(self, probes, active=False, hypothesis=None):
        self.probes=probes
        self.active=active
        self.hypothesis=hypothesis

    def __str__(self):
        return "active: {a}, hyp: {h}, probes: {p}".format(
            a=self.active,
            h=self.hypothesis,
            p=self.probes,)

ml_swarm = make_ml_swarm(
    layers=[
        (row_swarm,random_row), (col_swarm,random_col)
    ],
    agent_count=100,
)

def ml_diffuse_layer(layer):
    for probe in layer:
        ml_diffuse_probe(probe)

def ml_diffuse_probe(probe):
    if not probe.active:
        polled_probe = random.choice(layer)
        if polled_probe.active:
            probe.hypothesis = polled_probe.hypothesis
        else:
            for lower_probe, diffusion_function in probe.probes:
                diffusion_function(lower_probe)
            probe.hypothesis = tuple(lower_probe.hypothesis for lower_probe in self.probes)

print(ml_swarm[0])
print(ml_swarm[1])
print(row_swarm[:3])
print(col_swarm[:3])

def microtest(hyp):
    return search_space[hyp.row][hyp.col] > random.randint(0,10)

```

Uses Agent 5 and initialise 6a.

11 String search implemented with the library

```
68  <src/string-search.py 68>≡ (96c)
    import random
    import functools
    import sds

    search_space = "xxhellxelloxhexhelxxxxhxlloxxx"
    model = "hello"

    def random_hyp(rnd): return rnd.randint(0,len(search_space)-len(model))

    def make_microtest(offset):
        return lambda hyp: search_space[hyp+offset] == model[offset]

    microtests = [make_microtest(offset) for offset in range(len(model))]

    clusters = sds.run(
        swarm=sds.Agent.initialise(agent_count=1000),
        microtests=microtests,
        random_hypothesis_function=random_hyp,
        max_iterations=300,
        diffusion_function=sds.passive_diffusion,
        rng=random.Random(),
        report_iterations=10,
    )

    print(clusters.most_common())
    Uses Agent 5, initialise 6a, passive_diffusion 13, and run 35.
```

12 Stochastic Diffusion Sort implemented with the library

69

<src/sds-sort.py 69>≡

```
import functools
import random
import sds
```

```
search_space = [3, 9, 4, 1, 6, 1, 2, 2, 3, 2, 1, 7, 8, 1, 0, 0, 4, 8, ]
```

```
def random_hyp(rnd):
    return rnd.randrange(len(search_space))
```

```
def test_hyp(hyp):
    return -search_space[hyp]
```

```
microtests = [test_hyp]
```

```
agent_count = 1000
```

```
swarm = sds.Agent.initialise(agent_count)
```

```
clusters = sds.run(swarm, microtests, random_hyp, test_phase_function=sds.comparative_t
```

```
solution = [(search_space[hyp],count) for hyp, count in clusters.most_common()]
```

```
print(clusters.most_common(1))
```

```
print(solution)
```

Uses Agent 5, comparative_test_phase 9, initialise 6a, and run 35.

13 SDS Daemon implemented with the library

```
70  <src/sds-daemon.py 70>≡
    import functools
    import random
    import sds

    search_space, model = "xxhexlxelloxhexhxlxoxxxhxllxxx", "hello"

    def random_hyp(rnd):
        return rnd.randint(0, len(search_space) - len(model))

    def test_hyp(offset, hyp):
        return search_space[hyp + offset] == model[offset]

    microtests = [
        functools.partial( test_hyp, offset)
        for offset
        in range(len(model))
    ]

    swarm = sds.Agent.initialise(agent_count=1000)

    max_iterations = None

    diffusion_function=sds.passive_diffusion

    sds.run_daemon(
        swarm,
        microtests,
        random_hyp,
        diffusion_function,
        max_iterations,
        halting_function=sds.never_halt,
        halting_iterations=0,
        multitesting=1,
        multitest_function=all,
        report_iterations=100,
        test_phase_function=sds.test_phase,
        hypothesis_string_function=str,
        max_cluster_report=None,
        out_file_name='./daemon-clusters.json',
        random=random,
    )

    print('done src/sds-daemon.py')
```

Uses Agent 5, initialise 6a, never_halt 21b, passive_diffusion 13, run_daemon 39, and test_phase 8.

14 Data-driven SDS implemented with the library

This implements data driven sds. It has the problem that you can reach strong stability with imperfect hypotheses.

See this final state, where `model = 'hello'` and `hypothesis 6 == 'xello'`.

```
[(CompHyp(hyp=6, data=3), 381), (CompHyp(hyp=6, data=2), 296),  
 (CompHyp(hyp=6, data=4), 171), (CompHyp(hyp=6, data=1), 152),]  
[(6, 1000)]
```

This shows that all of the 1000 agents have converged on hypothesis 6. They are also maintaining the data points `[1,2,3,4]`, i.e. not 0, which is the only test which fails.

```

72  <src/data-driven.py 72>≡
    import random, functools, sds
    from collections import namedtuple, Counter

    search_space, model = "xxhexlxelloxhexhxlxoxxxhxlxxxx", "hello"

    CompHyp = namedtuple('CompHyp', ('hyp', 'data'))

    def make_microtest(offset):
        return lambda hyp: search_space[hyp+offset] == model[offset]

    microtests = [make_microtest(offset) for offset in range(len(model))]

    def random_hyp(rnd): return rnd.randint(0, len(search_space)-len(model))

    def data_driven_random_hyp(rnd):
        return CompHyp(
            hyp=random_hyp(rnd),
            data=rnd.randrange(len(microtests)),)

    def data_driven_microtest(swarm, compound_hyp):
        random_agent = random.choice(swarm)

        result = microtests[random_agent.hypothesis.data](compound_hyp.hyp)

        random_agent.active = result

        return result

    swarm = sds.Agent.initialise(agent_count=1000)

    clusters = sds.run(
        swarm,
        microtests=[functools.partial(data_driven_microtest, swarm)],
        random_hypothesis_function=data_driven_random_hyp,
        max_iterations=300,
        diffusion_function=sds.passive_diffusion,
        random=random,
    )

    no_data_clusters = Counter()
    [no_data_clusters.update({hyp:count}) for (hyp,data), count in clusters.items()]
    print(clusters.most_common(), no_data_clusters.most_common())
    Uses Agent 5, initialise 6a, passive_diffusion 13, and run 35.

```


15 Coupled SDS implemented with the library

```
73  <src/coupled-sds.py 73>≡
    import itertools, random, sds

    swarms = [
        sds.Agent.initialise(agent_count=100),
        sds.Agent.initialise(agent_count=101),]

    search_space, model = "xxhexlxlloxxhexhxlxxxxhllxxxx","hello"

    microtests = [ lambda loc,data: search_space[loc+data] == model[data] ]

    random_hypothesis_functions = [
        lambda rnd: rnd.randint(0,len(search_space)-len(model)),
        lambda rnd: rnd.randrange(len(model)),
    ]

    diffusion_functions = (
        sds.passive_diffusion,
        sds.context_sensitive_diffusion,)

    test_phase_functions = (
        sds.synchronous_coupled_test_phase,
        sds.sequential_coupled_test_phase,)

    is_comparative = (False, True)

    if is_comparative:
        multitest_function = max
    else:
        multitest_function = all

    for test_phase_function, compare in itertools.product(
        test_phase_functions,
        is_comparative,
    ):

        clusters = sds.run_coupled(
            swarms=swarms,
            microtests=microtests,
            random_hypothesis_functions=random_hypothesis_functions,
            max_iterations=100,
            diffusion_functions=diffusion_functions,
            random=random,
            multitesting=1,
            multitest_function=multitest_function,
            report_iterations=None,
            test_phase_function=test_phase_function,
```

```
        hypothesis_string_function=None,
        max_cluster_report=None,
        compare=compare,
    )

    print(clusters)
Uses Agent 5, context_sensitive_diffusion 15, initialise 6a, passive_diffusion 13,
run_coupled 48, sequential_coupled_test_phase 46, and synchronous_coupled_test_phase 45.
```

16 SDS Simulator implemented with the library

```
75  <sds simulator 75>≡ (91)
    def simulate(
        scores,
        max_iterations=1000,
        report_iterations=500,
        diffusion_function=passive_diffusion,
        agent_count=1000,
        multitesting=1,
        multitest_function=all,
        random=random,
        random_hyp=None,
        halting_function=never_halt,
        halting_iterations=None,
    ):

        if random_hyp is None:

            def random_hyp(rnd): return rnd.randrange(1,len(scores))

        if halting_iterations is None:
            halting_iterations = report_iterations

        def make_microtest(test_num, rnd):
            return lambda hyp: rnd.random() < scores[test_num]

        microtests = [
            lambda hyp: random.random() < scores[hyp]
        ]

        swarm=Agent.initialise(agent_count=agent_count)

        swarm[0].active = True
        swarm[0].hypothesis = 0

        clusters = run(
            swarm=swarm,
            microtests=microtests,
            random_hypothesis_function=random_hyp,
            max_iterations=max_iterations,
            diffusion_function=passive_diffusion,
            multitesting=multitesting,
            multitest_function=multitest_function,
            random=random,
            report_iterations=report_iterations,
            halting_function=halting_function,
            halting_iterations=halting_iterations,
        )
```

Uses Agent 5, initialise 6a, never_halt 21b, passive_diffusion 13, and run 35.

(91) $\triangleleft 63b$

16.1 SDS Simulator front end

```
77  <src/simulator.py 77>≡
    import random
    import functools
    import sds
    import argparse

    if __name__ == '__main__':

        parser = argparse.ArgumentParser(
            description="Simulates SDS",
            epilog="",
        )

        parser.add_argument(
            "-s", "-scores",
            nargs='+',
            required=True,
            type=float,
            help="Scores of the hypotheses",
        )

        parser.add_argument(
            "-e", "-seed",
            type=int,
            help="Random seed",
        )

        parser.add_argument(
            "-i", "-iterations",
            type=int,
            default=1000,
            help="Maximum iterations",
        )

        parser.add_argument(
            "-r", "-report-iterations",
            type=int,
            default=None,
            help="Number of iterations between every analysis",
        )

        parser.add_argument(
            "-n", "-agent-count",
            type=int,
            default=1000,
            help="Number of agents in the swarm",
        )
```

```

diffusion_types = [
    'passive',
    'context-free',
    'context-sensitive',
]
diffusion_functions = [
    sds.passive_diffusion,
    sds.context_free_diffusion,
    sds.context_sensitive_diffusion,
]

parser.add_argument(
    "-d", "-diffusion",
    choices=diffusion_types,
    default=diffusion_types[0],
    help="Type of diffusion phase",
)

parser.add_argument(
    "-p", "-prob",
    default=0,
    type=str,
    help="Probability of picking optimal hypothesis",
)

halting_types = [
    'never',
    'weak',
    'threshold',
]
halting_functions = [
    sds.never_halt,
    sds.make_stability_halting_function,
    sds.make_instant_threshold_halt_function,
]
parser.add_argument(
    '-a', '-halt',
    choices=halting_types,
    default=halting_types[0],
    help='Type of halting',
)
parser.add_argument(
    '-l', '-lower',
    type=float,
    default=0.5,
    dest='halting_lower_bound',
    help="Activity lower bound for stability halting",
)

```

```

parser.add_argument(
    '-g', '-region',
    type=float,
    default=0.25,
    dest='halting_stability_region',
    help="Activity stability region for stability halting",
)
parser.add_argument(
    '-t', '-time',
    type=int,
    default=32,
    dest='halting_stability_time',
    help="Activity stability time for stability halting",
)

args = parser.parse_args()

print(args)

def random_hyp_fun(P, score_len, rnd):
    if rnd.random() < P:
        return 0
    else:
        return rnd.randrange(1, score_len)

if args.prob == 'even':
    args.random_hyp = lambda rnd: rnd.randrange(len(args.scores))
else:
    args.random_hyp = lambda rnd: random_hyp_fun(float(args.prob), len(args.scores),

if args.report_iterations is None:
    args.report_iterations = args.iterations // 10

args.diffusion = dict(
    zip(diffusion_types, diffusion_functions)
)[args.diffusion]

halting_dict = dict(zip(halting_types, halting_functions))
halting_function = halting_dict[args.halt]
if args.halt == 'never':
    pass
elif args.halt == 'weak':
    halting_function = halting_function(
        lower=args.halting_lower_bound,
        region=args.halting_stability_region,
        time=args.halting_stability_time,)
elif args.halt == 'threshold':
    halting_function = halting_function(

```

```

        threshold=args.halting_lower_bound,)

print(args)

clusters = sds.simulate(
    args.scores,
    random=random.Random(args.seed),
    max_iterations=args.iterations,
    report_iterations=args.report_iterations,
    diffusion_function=args.diffusion,
    agent_count=args.agent_count,
    random_hyp=args.random_hyp,
    halting_function=halting_function,
)

print(clusters.most_common())

```

Uses context_free_diffusion 14, context_sensitive_diffusion 15, make_instant_threshold_halt_function 23, make_stability_halting_function 22, never_halt 21b, passive_diffusion 13, and simulate 75.

17 Heterogeneous agents

```
81  <src/heterogeneous-sds.py 81>≡
    import random, functools, sds
    from collections import namedtuple

    def random_hypothesis_function(random):
        return random.randint(0,100)

    def test_is_odd(hypothesis):
        return hypothesis % 2 == 1

    def test_is_even(hypothesis):
        return hypothesis % 2 == 0

    def test_is_large(hypothesis):
        return random.randint(0,100) < hypothesis

    def test_is_small(hypothesis):
        return not test_is_large(hypothesis)

    def is_prime(hypothesis):
        return hypothesis in (2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,

    def is_square(hypothesis):
        return hypothesis in (1, 4, 9, 16, 25, 36, 49, 64, 81, 100)

    TestList = namedtuple('TestList',('last_agent','tests'))

    # last_agent refers to the number of the first agent which does not use
    # the tests.

    microtests = [
        TestList(last_agent=1000,tests=[
            is_square,
            test_is_small,
            test_is_even,
        ]),
        TestList(last_agent=2000,tests=[
            test_is_large,
            is_prime,
        ]),
    ]

    def heterogeneous_test_phase(swarm, microtests, random, multitesting=1, multitest_function=None):

        if multitest_function is None:
            if compare:
                multitest_function = max
```

```

        else:
            multitest_function = all

def zip_agents_and_tests(swarm, microtests):

    prev_last_agent = 0

    for last_agent_num, test_list in microtests:

        yield (swarm[prev_last_agent:last_agent_num], test_list)

        prev_last_agent = last_agent_num

    for (
        partial_swarm,
        partial_microtests,
    ) in (
        zip_agents_and_tests(swarm, microtests)
    ):

        for _ in sds.generic_test_phase(
            partial_swarm,
            partial_microtests,
            random,
            multitesting,
            multitest_function,
            compare,
        ):
            pass

agent_count = 2000
swarm = sds.Agent.initialise(agent_count)

max_iterations = 4000
diffusion_function = sds.context_sensitive_diffusion
#diffusion_function = sds.passive_diffusion

clusters = sds.run(
    swarm,
    microtests,
    random_hypothesis_function,
    max_iterations,
    diffusion_function,
    random,
    test_phase_function=heterogeneous_test_phase,
)

print(clusters.most_common(10))

```

Uses Agent 5, context_sensitive_diffusion 15, generic_test_phase 10, initialise 6a,
passive_diffusion 13, and run 35.

18 Intransitive Dice

```
84  <src/sds-intransitive-dice.py 84>≡
    import random
    import sds

    # A > B > C. In a rumble, B wins most often, A and C win equally
    highest = (
        (2,2,6,6,7,7),
        (1,1,5,5,9,9),
        (3,3,4,4,8,8),
    )

    # A > B > C. In a rumble, C wins least often, A and B win equally
    lowest = (
        (2,2,4,4,9,9),
        (1,1,6,6,8,8),
        (3,3,5,5,7,7),
    )

    # A > B > C > D > A
    efrons = (
        (4,4,4,4,0,0),
        (3,3,3,3,3,3),
        (6,6,2,2,2,2),
        (5,5,5,1,1,1),
    )

    # Uses all numbers 1-24, functionally equivalent to efrons
    efrons24 = (
        (1,2,16,17,18,19),
        (10,11,12,13,14,15),
        (6,7,8,9,23,24),
        (3,4,5,20,21,22),
    )

    # All dice have an equal mean average roll.
    eqavg = (
        (7,7,7,7,1,1),
        (5,5,5,5,5,5),
        (9,9,3,3,3,3),
        (8,8,8,2,2,2),
    )

    # the probability that M3 rolls a higher number than M4 is 17/36
    # the probability that M4 rolls a higher number than M5 is 17/36
    # the probability that M5 rolls a higher number than M3 is 17/36
    # 17/36 == 0.4722222222
    miwin = (
```

```

        (1,2,5,6,7,9),
        (1,3,4,5,8,9),
        (2,3,4,6,7,8),
    )

    # Slight deviant from traditional
    deviant = (
        (1,1,3,5,5,6),
        (2,3,3,4,4,5),
        (1,2,2,4,6,6),
    )

    extreme = (
        (7,7,7,7,-100,-100),
        (5,5,5,5,5,5),
        (1000000,1000000,3,3,3,3),
        (8,8,8,2,2,2),
    )

    dice = extreme

    def new_hyp(rng=random):
        return rng.randrange(len(dice))

    def microtest(hyp):
        return random.choice(dice[hyp])

    swarm=sds.Agent.initialise(1000)

    for i in range(1000):
        clusters = sds.run(
            swarm=swarm,
            max_iterations=1,
            random_hypothesis_function=new_hyp,
            diffusion_function=sds.context_free_diffusion,
            microtests=[microtest],
            test_phase_function=sds.comparative_test_phase,
            report_iterations=None,)

        print(clusters.most_common())

```

Uses Agent 5, comparative_test_phase 9, context_free_diffusion 14, initialise 6a, and run 35.

19 True Restaurant Game

```
86  <src/restaurant-game.py 86>≡
    import random, functools, sds
    from collections import namedtuple
    from pprint import pprint

    cuisine_count = 3

    restaurant_count = 10

    agent_count = 1000

    swarm = sds.Agent.initialise(agent_count)

    microtests = None

    max_iterations = 10000

    diffusion_function = sds.passive_diffusion

    def clip(num, smallest, largest):
        return min(largest, max(num, smallest))

    def make_restaurant(cuisine_count):

        return sorted(
            [0] + [
                random.random()
                for _
                in range(cuisine_count-1)
            ]
        )

    restaurants = [
        make_restaurant(cuisine_count)
        for restaurant_num
        in range(restaurant_count)
    ]

    def to_probabilities(r):
        return [t-p for p,t in zip(r,r[1:] + [1])]

    #pprint([to_probabilities(r) for r in restaurants])

    def generic_random_hypothesis_function(restaurant_count, random):

        return random.randrange(restaurant_count)
```

```

random_hypothesis_function = functools.partial(
    generic_random_hypothesis_function,
    restaurant_count)

def make_taste(cuisine_count):

    general_happiness = random.gauss(0,1)

    return [
        clip(random.gauss(2/3,0.5)+general_happiness,smallest=0,largest=1)
        for _
        in range(cuisine_count)
    ]

    #taste = [1] * cuisine_count

    #taste[random.randrange(cuisine_count)] = 0

    return taste

tastes = [
    make_taste(agent_count)
    for agent_num
    in range(agent_count)
]

def pick_dish(restaurant):

    num = random.random()

    return sum(1 for x in restaurant if num >= x)

def taste_test(restaurant, agent_tastes):

    dish = pick_dish(restaurant)

    agent_taste = agent_tastes[dish]

    return random.random() < agent_taste

def exhaustive_test(tastes, restaurants):

    restaurant_scores = []

    for r_num, restaurant in enumerate(restaurants):
        probs = to_probabilities(restaurant)
        agent_scores = []
        for taste in tastes:

```

```

        agent_scores.append(
            sum(dish*prob for dish,prob in zip(taste,probs))
        )
        restaurant_score = sum(agent_scores)/len(tastes)
        restaurant_scores.append(
            (r_num, restaurant_score)
        )

    return restaurant_scores

def restaurant_game_test_phase(
    swarm,
    microtests,
    random,
    **kwargs
):

    for agent_num, agent in enumerate(swarm):

        agent_tastes = tastes[agent_num]

        agent.active = taste_test(
            restaurants[agent.hypothesis],
            agent_tastes)

clusters = sds.run(
    swarm,
    microtests,
    random_hypothesis_function,
    max_iterations,
    diffusion_function,
    random,
    report_iterations=1000,
    test_phase_function=restaurant_game_test_phase,
)

print(clusters)

truth = exhaustive_test(tastes, restaurants)

scores = [score for num,score in truth]

for hyp, count in clusters.most_common():
    print("Restaurant {hyp} has {c} agents, and a score of {s}".format(
        hyp=hyp,
        c=count,
        s=scores[hyp],
    ))

```



```
truth.sort(key=lambda x:-x[1])  
print(truth[:10])
```

Uses Agent 5, initialise 6a, passive_diffusion 13, and run 35.

20 Library definition

```
90 <src/library/setup.py 90>≡
    from setuptools import setup

    setup(
        name='sds',
        version='0.1.5',
        packages=['sds'],
        description='Stochastic Diffusion Search',
        keywords = ['swarm', 'artificial', 'intelligence', 'search'],
        classifiers = [
            'Development Status :: 3 - Alpha',
            'Intended Audience :: Developers',
            'Intended Audience :: Science/Research',
            'License :: OSI Approved :: Apache Software License',
            'Operating System :: OS Independent',
            'Programming Language :: Python :: 3',
            'Programming Language :: Python',
            'Topic :: Scientific/Engineering :: Artificial Intelligence',
        ],
        url='http://www.aomartin.co.uk/sds-library/',
        author='Andrew Owen Martin',
        author_email='a.martin@gold.ac.uk',
        long_description="""\
A library which implements the main variants of Stochastic Diffusion
Search (SDS), and provides a convenient front end.

Stochastic Diffusion Search (SDS) is a generic population-based search
method. SDS agents perform cheap, partial evaluations of a hypothesis (a
candidate solution to the search problem). They then share information
about hypotheses (diffusion of information) through direct one-to-one
communication. As a result of the diffusion mechanism, high-quality
solutions can be identified from clusters of agents with the same
hypothesis.

This is a library used during the writing of my PhD thesis, I will
publish full documentation and host the code on GitHub once the design
has settled down and I have submitted my thesis. Until then, feel free
to email me.

SDS has a Scholarpedia page:
http://www.scholarpedia.org/article/Stochastic\_diffusion\_search

A list of papers written on SDS can be found in the Stochastic Diffusion
Search paper repository, maintained by the author of this module:
http://www.aomartin.co.uk/sds-repository/publications.html
"""
    )
```

91 `<src/library/sds/__init__.py 91>≡`
 `<library dependencies 7b>`
 `<agent class definition 5>`
 `<agent namedtuple 7a>`
 `<generic test phase function 10>`
 `<test phase function 8>`
 `<comparative test phase function 9>`
 `<generic single agent test function 12>`
 `<generic diffusion function 18>`
 `<generic single agent diffusion function 19>`
 `<passive diffusion function 13>`
 `<context free diffusion function 14>`
 `<context sensitive diffusion function 15>`
 `<synchronous iterate function 27>`
 `<asynchronous iterate function 29>`
 `<parallel iterate function 31>`
 `<parallel update state function 33>`
 `<never halt function 21b>`
 `<stability halt function 22>`
 `<instant threshold halt function 23>`
 `<threshold time halt function 24>`
 `<run function 35>`
 `<handle halting function 25>`
 `<basic report function 26a>`
 `<handle reporting function 26b>`
 `<run daemon 39>`
 `<coupled diffusion 41b>`
 `<coupled test phase 42>`
 `<synchronous coupled test phase 45>`
 `<sequential coupled test phase 46>`
 `<coupled iterate 47>`
 `<coupled run 48>`
 `<count clusters function 49>`
 `<write swarm function 37b>`
 `<activity function 50b>`
 `<estimate noise 51>`
 `<swarm from clusters 52>`
 `<pretty print clusters with values 53>`
 `<sds simulator 75>`
 `<swarm class 54>`
 `<make multilayer sds function 60>`
 `<single diffusion function 62a>`
 `<random compound hypothesis function 62b>`
 `<flatten hypothesis 63a>`

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```
96a  <src/library/MANIFEST.in 96a>≡
      include LICENSE.txt
      include doc/string_search.py
      include doc/documentation.pdf
      include doc/tutorial.html
      include doc/tutorial.pdf

96b  <src/library/README.rst 96b>≡
      This is the readme for the sds module.

96c  <src/library/doc/string-search.py 96c>≡
      <src/string-search.py 68>
```

20.1 Deploying to PyPi

I deployed this to PyPi by going into `thesis/src/library` and running `python3 setup.py sdist upload`.

I deployed an update by updating the version line in `<src/library/setup.py 90>`, running `make pip_install` and running `python setup.py sdist upload` in the virtual environment.

21 Mathematical library definition

```
96d  <src/mathlib/setup.py 96d>≡
      from setuptools import find_packages, setup

      setup(
          name='sdsmath',
          version='0.1.1',
          packages=find_packages(),
          url='www.aomartin.co.uk',
          author='Andrew Owen Martin',
          author_email='a.martin@gold.ac.uk',
      )
```


97a $\langle \text{src/mathlib/sdsmath/sdsmath.py 97a} \rangle \equiv$
 $\langle \text{math library dependencies 97b} \rangle$
 $\langle \text{diffusion rate model (never defined)} \rangle$
 $\langle \text{basic model (never defined)} \rangle$
 $\langle \text{minimum stability model (never defined)} \rangle$

97b $\langle \text{math library dependencies 97b} \rangle \equiv$ (97a)
from sympy import Symbol as _Symbol
from sympy import Eq as _Eq
from sympy import solveset as _solveset
from sympy import latex as _latex
from sympy import simplify as _simplify

97c $\langle \text{src/mathlib/sdsmath/__init__.py 97c} \rangle \equiv$
from .sdsmath import *

97d $\langle \text{src/mathlib/LICENSE.py 97d} \rangle \equiv$

97e $\langle \text{src/mathlib/MANIFEST.in 97e} \rangle \equiv$

97f $\langle \text{src/mathlib/README.rst 97f} \rangle \equiv$
This is the readme for the sdsmath module.

21.1 Interacting Markov Chain Model

This is the interacting markov chain model implemented in Sympy.

```
98  <interacting markov chain model 98>≡
    import sympy

    pminus, pm, N = sympy.symbols('p^{-} p_{m} N')

    a = 2*(1-pminus)*(1-pm)
    pi1 = (a-1 + sympy.sqrt( pow(a-1,2) + 2*a*pm*(1-pminus) ) ) / a
    pi2 = (1 - sympy.sqrt( pow(a-1,2) + 2*a*pm*(1-pminus) ) ) / a

    n = ((N+1)*pi1-1, (N+1)*pi1)

    En = N*pi1

    sigma = sympy.sqrt(N * pi1 * pi2)

    def interacting_markov_chain_model(pminus_val, pm_val, N_val):
        subs = [(N,N_val),(pm,pm_val),(pminus,pminus_val)]
        ngeq, nleq = (n_eq.subs(subs) for n_eq in n)

        return {
            'n >=':ngeq,
            'n <=':nleq,
            'E[n]':En.subs(subs),
            'sigma':sigma.subs(subs),
        }
```